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$N = 1$ supergravity in Euclidean 11D in terms of graded pseudo-Majorana spinors

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Abstract

We study the complex-conjugation and flipping properties of the recently discovered *graded* pseudo-Majorana (GPM) spinors in general d-dimensional spacetime with the signature $((-)^t, (+)^s)$. We consider simple supergravity in Euclidean eleven dimensions (E11D) as an example. We show that we can formulate $N = 1$ GPM supergravity with 128 + 128 degrees of freedom in E11D with no doubling of supersymmetries. Unlike the conventional formulation with symplectic $Sp(n)$ spinors, no doubling is needed for GPM spinors. Similar properties are also found for $N = 1$ GPM supergravity in Euclidean 4D.

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1. Introduction

Supersymmetry or supergravity in Euclidean dimensions has drawn much attention recently. For example, in the recent formulation of non-commutative $N = 1/2$ superspace [1] or $N = (1, 1)$ superspace [2], Euclidean spacetime is crucial for the θ coordinates to have certain non-vanishing anti-commutator. Moreover, as explained in [2], it is necessary to use 'pseudo-conjugation' instead of ordinary conjugation in supersymmetric theories in Euclidean spacetime. This recent development gives us a strong motivation of investigating supergravity in diverse Euclidean dimensions, such as Euclidean eleven dimensions (E11D) or Euclidean four dimensions (E4D).

In the conventional analysis of spinors in diverse dimensions [3–5], there exist only symplectic pseudo-Majorana spinors in E11D[1]. This implies that the simplest (pseudo) Majorana spinors in E11D have at least $32 = 2 \times 16$ components in the **2** of $Sp(1)$. Therefore, the number of supersymmetries doubles, so that the minimal number of supersymmetries is

[1] In our present paper, we use the signature $(+, +, \cdots, +)$ as Euclidean 11D.

$N = 2$ instead of $N = 1$ in E11D. This is problematic for the possible Euclidean version of 11D supergravity [6], because according to [3], there is no simple $N = 1$ supersymmetry with 128 + 128 degrees of freedom corresponding to the conventional $N = 1$ supergravity in Minkowskian 11D (M11D) [6][2].

Independently of this issue, it has recently been pointed out [7] that by introducing the new concept of 'graded Majorana' (GM) or 'graded pseudo-Majorana' (GPM) spinors with the concept of 'pseudo-conjugation' we do not need to double the number of spinors. As a matter of fact, the importance of pseudo-conjugation in Euclidean supersymmetry was first pointed out back in 1980s [8]. We clarify this concept of GM or GPM related to pseudo-conjugation by reviewing first the conventional formulation [3–5].

Consider any arbitrary spacetime dimensions $d = t + s$ with the signature $(\eta_{00}, \eta_{11}, \ldots, \eta_{t-1,t-1}, \ldots, \eta_{d-1,d-1}) = ((-)^t, (+)^s) = (\underbrace{-, -, \ldots, -}_{t}, \underbrace{+, +, \ldots, +}_{s})$, following the notation in [4]. There are three matrices A, B and C in general $d = t + s$ dimensions [3, 4]. The matrix A is defined by the products of the γ-matrices $\gamma_0, \gamma_1, \ldots, \gamma_{t-1}$ for time coordinates associated with the Dirac conjugation by $\bar{\psi} \equiv \psi^\dagger A$. The matrix B is associated with the complex conjugation of the γ-matrices as specified in (1.1c), while C is the charge conjugation matrix associated with $\bar{\psi} = \psi^T C$ for (pseudo) Majorana spinor ψ [4]. There are two parameters $\epsilon = \pm 1$ and $\eta = \pm 1$, associated with all of these relationships summarized as [4][3]

$$\bar{\psi} \equiv \psi^\dagger A = \epsilon \psi^T C, \qquad \psi^* \equiv B\psi, \tag{1.1a}$$

$$(\gamma_\mu)^\dagger = (-1)^t A \gamma_\mu A^{-1}, \qquad A \equiv \gamma_0 \gamma_1 \cdots \gamma_{t-1}, \tag{1.1b}$$

$$(\gamma_\mu)^* = \eta B \gamma_\mu B^{-1}, \qquad B^* B = \epsilon I, \qquad B^T = \epsilon B, \tag{1.1c}$$

$$(\gamma_\mu)^T = (-1)^t \eta C \gamma_\mu C^{-1}, \qquad C^\dagger C = I, \tag{1.1d}$$

$$C^T = \epsilon \eta^t (-1)^{t(t+1)/2} C, \qquad C \equiv BA. \tag{1.1e}$$

In the conventional formulation [3, 4], it is clear that the case $\epsilon = -1$ with

$$B^* B = -I \tag{1.2}$$

has a problem, because $(\psi^*)^* = (B\psi)^* = B^*\psi^* = B^*B\psi = -\psi \Longrightarrow (\psi^*)^* = -\psi$. The conventional way to avoid this problem is to double the number of components of ψ by introducing symplectic spinors [3–5]. In fact, by introducing ψ_i $(i, j = 1, 2)$ in the **2** of $Sp(1)$ [3–5], its complex conjugation is

$$\psi^{*i} \equiv (\psi_i)^* = B\epsilon^{ij}\psi_j, \tag{1.3}$$

so that we can show $[(\psi_i)^*]^* = +\psi_i$ by

$$[(\psi_i)^*]^* = (B\epsilon^{ij}\psi_j)^* = B^*\epsilon_{ij}(\psi_j)^* = B^*\epsilon_{ij}(B\epsilon^{jk}\psi_k) = B^*B\big(-\delta_i{}^k\big)\psi_k = +\psi_i, \tag{1.4}$$

thanks to the properties of the Sp(1) metric $\epsilon_{ij}\epsilon^{jk} = -\delta_i{}^k$, $(\epsilon^{ij})^* = \epsilon^{ij} = \epsilon_{ij}$ and (1.2).

However, the new discovery in [7] is that if we allow the so-called pseudo-conjugation operator $\diamond$ for GM or GPM spinor ψ satisfying $(\psi^\diamond)^\diamond = -\psi$, we need no symplectic doubling, because $\psi^\diamond = B\psi$ leads to $(\psi^\diamond)^\diamond = (B\psi)^\diamond = B^*\psi^\diamond = B^*B\psi = -\psi$:

$$\psi^\diamond \equiv B\psi \qquad \Longrightarrow \qquad (\psi^\diamond)^\diamond = -\psi. \tag{1.5}$$

Compared with the conventional prescription [3, 4], GM or GPM spinors with the $\diamond$-conjugations [7] are much simpler, with no need of symplectic doublings.

[2] In our paper, M11D have the signature $(+, -, -, \cdots, -)$ as in [6].

[3] There is a typographical error in equation (3) in [4]. The factor (-1) in there should be replaced by $(-1)^t$.

Table 1. GM and GPM spinors in diverse dimensions.

ϵ	η	$s-t$	Spinors	Pseudo-conjugations
-1	$+1$	4, 5, 6 (mod 8)	GM	$\psi^\diamond = B\psi$
-1	-1	2, 3, 4 (mod 8)	GPM	$\psi^\diamond = B\psi$

2. GM and GPM spinors in diverse dimensions

Before considering the special case of E11D, we first establish the general aspects of GM or GPM spinors in arbitrary spacetime dimensions $^\forall d = t + s$ with the signature $((-)^t, (+)^s)$ [4], using the general formulae (1.1). As has been mentioned, we need either GM or GPM spinors, iff $\epsilon = -1$. All these possible cases are given in table 1: this table is complementary to the table in [4] for GM or GPM spinors.

The general complex-conjugation and flipping properties for GM or GPM spinors are

$$(\widetilde{\psi}\gamma^{[n]}\chi)^\diamond = +\eta^{n+t}(\widetilde{\psi}\gamma^{[n]}\chi), \tag{2.1a}$$

$$(\widetilde{\psi}\gamma^{[n]}\chi) = +\eta^{n+t}(-1)^{(n-t)(n-t-1)/2}(\widetilde{\chi}\gamma^{[n]}\psi). \tag{2.1b}$$

Here our Dirac conjugation $\widetilde{\psi}$ is defined by

$$\widetilde{\psi} \equiv (\psi^\diamond)^T A, \qquad \psi^\diamond = B\psi, \tag{2.2}$$

where the conventional complex conjugation in $\psi^\dagger \equiv (\psi^*)^T$ is replaced by that with a 'pseudo-conjugation' $(\psi^\diamond)^T$ [7], to be distinguished from the former. The $\gamma^{[n]}$ is for a totally antisymmetric product of n γ-matrices, e.g., $\gamma^{[3]}$ is equivalent to $\gamma^{\mu\nu\rho}$. Equation (2.1b) is the same as equation (6) in [4] for $\epsilon = -1$, whenever GM or GPM spinors exist, because this flipping property is common to graded and non-graded (pseudo) Majorana spinors.

Equation (2.1a) is confirmed as follows. First, the equivalence between $\diamond$ and $*$-operations on bosonic quantities [7] makes the first equality trivial. For the second equality, we need

$$A^\diamond = A^* = \eta^t BAB^{-1}, \qquad (\gamma^{[n]})^\diamond = (\gamma^{[n]})^* = \eta^n B\gamma^{[n]}B^{-1}, \tag{2.3a}$$

$$A^T = (-1)^{t(t+1)/2}\eta^t CAC^{-1}, \qquad (B^T)^{-1} = \epsilon AC^{-1}, \qquad \psi^T = \widetilde{\psi}A^{-1}(B^{-1})^T, \tag{2.3b}$$

which are easily obtained from (1.1) or (2.2). Using (2.3), we confirm (2.1) as

$$\begin{aligned}
(\text{LHS of 2.1a}) &= (\widetilde{\psi}\gamma^{[n]}\chi)^\diamond = [(\psi^\diamond)^T A\gamma^{[n]}\chi]^\diamond = -\psi^T A^*(\gamma^{[n]})^*\chi^\diamond \\
&= -[\widetilde{\psi}A^{-1}(\epsilon AC^{-1})](\eta^t BAB^{-1})(\eta^n B\gamma^{[n]}B^{-1})(B\chi) \\
&= -\eta^{t+n}\epsilon(\widetilde{\psi}C^{-1}BA\gamma^{[n]}\chi) = +\eta^{t+n}[\widetilde{\psi}C^{-1}(CA^{-1})A\gamma^{[n]}\chi] \\
&= +\eta^{n+t}(\widetilde{\psi}\gamma^{[n]}\chi) = (\text{RHS of 2.1a}).
\end{aligned} \tag{2.4}$$

The most crucial relationship used is (1.2).

Note that equation (2.1) is general enough to be applied to $^\forall d = t + s$, as long as $s - t = 2, \ldots, 6$ (mod 8) for $\epsilon = -1$, when GM ($\eta = +1$) or GPM ($\eta = -1$) spinors exist.

3. $N = 1$ GPM supergravity in E11D

We are now ready to look into the special case of E11D, implying that $s = 11, t = 0, s - t = 11 = 3$ (mod 8) and $\epsilon = -1, \eta = -1$, as in our table 1. In E11D, equation (1.1) is more specified as

$$(\gamma_\mu)^\dagger = \gamma_\mu, \qquad A = I, \qquad \widetilde{\psi} \equiv (\psi^\diamond)^T, \qquad \psi^\diamond = B\psi, \tag{3.1a}$$

$$(\gamma_\mu)^* = (\gamma_\mu)^\diamond = -B\gamma_\mu B^{-1}, \qquad B = CA^{-1} = C, \qquad (B^T)^{-1} = -C^{-1}, \tag{3.1b}$$

$$(\gamma_\mu)^T = -C\gamma_\mu C^{-1}, \qquad C^T = -C. \tag{3.1c}$$

The condition $t = 0$ resulted in $A = I$. The most crucial lemmas are (2.1) now for $\epsilon = -1, \eta = -1, t = 0$:

$$(\widetilde{\psi}\gamma^{[n]}\chi)^\diamond = +(-1)^n(\widetilde{\psi}\gamma^{[n]}\chi), \tag{3.2a}$$

$$(\widetilde{\psi}\gamma^{[n]}\chi) = (-1)^{n(n+1)/2}(\widetilde{\chi}\gamma^{[n]}\psi). \tag{3.2b}$$

Therefore, the bilinears $(\widetilde{\psi}\chi), i(\widetilde{\psi}\gamma^\mu\chi), (\widetilde{\psi}\gamma^{\mu\nu}\chi), i(\widetilde{\psi}\gamma^{\mu\nu\rho}\chi), \ldots, i(\widetilde{\psi}\gamma^{[11]}\chi)$ are all real.

As careful readers may have noted, this property (3.2) is formally the same as that for $N = 1$ supergravity in M11D [6]. This has a great advantage, because we can directly use the results in M11D for our E11D, without further complications resulting from spinorial differences. This is one of the most important consequences of using GPM spinors, indicating the naturalness and validity of these spinors in E11D, which otherwise did not have such direct correspondence with the conventional M11D [6].

In the light of aforementioned preliminaries, we can present the Lagrangian for $N = 1$ GPM supergravity action $I_{\text{E11D}} \equiv \int d^{11}x \mathcal{L}_{\text{E11D}}$ in E11D:

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{E11D}} = &-\frac{1}{4}R(\omega) - \frac{i}{2}\left[\widetilde{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu\left(\frac{\omega+\hat{\omega}}{2}\right)\psi_\rho\right] - \frac{1}{48}(F_{\mu\nu\rho\sigma})^2 \\ &+ \frac{1}{192}(\widetilde{\psi}_\mu\gamma^{[\mu}\gamma_{\rho\sigma\tau\lambda}\gamma^{\nu]}\psi_\nu)(F^{\rho\sigma\tau\lambda} + \hat{F}^{\rho\sigma\tau\lambda}) + \frac{2}{(144)^2}\, e^{-1}\epsilon^{[4][4]'[3]}F_{[4]}F_{[4]'}A_{[3]}. \end{aligned} \tag{3.3}$$

All the relevant definitions are parallel to those in [6], such as

$$D_\mu(\omega)\psi_\nu \equiv \partial_\mu\psi_\nu + \frac{1}{4}\omega_\mu{}^{rs}(\gamma_{rs}\psi_\nu), \tag{3.4a}$$

$$\omega_\mu{}^{rs} \equiv \omega_\mu{}^{rs}(e) + K_\mu{}^{rs}, \qquad \hat{\omega}_\mu{}^{rs} \equiv \omega_\mu{}^{rs} + \frac{i}{4}(\widetilde{\psi}_\rho\gamma_\mu{}^{rs\rho\sigma}\psi_\sigma), \tag{3.4b}$$

$$K_\mu{}^{rs} \equiv -\frac{i}{4}(\widetilde{\psi}_\rho\gamma_\mu{}^{rs\rho\sigma}\psi_\sigma) - i(\widetilde{\psi}_\mu\gamma^{[r}\psi^{s]}) - \frac{i}{2}(\widetilde{\psi}^r\gamma_\mu\psi^s). \tag{3.4c}$$

Our action I_{E11D} is invariant under $N = 1$ GPM supersymmetry

$$\delta_Q e_\mu{}^m = -i(\widetilde{\epsilon}\gamma^m\psi_\mu), \tag{3.5a}$$

$$\delta_Q\psi_\mu = +D_\mu(\hat{\omega})\epsilon + \frac{i}{144}(\gamma_\mu{}^{\nu\rho\sigma\tau} - 8\delta_\mu{}^\nu\gamma^{\rho\sigma\tau})\epsilon\hat{F}_{\nu\rho\sigma\tau}, \tag{3.5b}$$

$$\delta_Q A_{\mu\nu\rho} = +\frac{3}{2}(\widetilde{\epsilon}\gamma_{[\mu\nu}\psi_{\rho]}). \tag{3.5c}$$

By definition, not only ψ_μ, but also ϵ are GPM spinors [7]. As is usual in supergravity [9], all the *hatted* quantities are supercovariant, e.g., $\hat{\omega}_\mu{}^{rs}$ in (3.4*b*), or

$$\hat{F}_{\mu\nu\rho\sigma} \equiv 4\partial_{[\mu}A_{\nu\rho\sigma]} - 3(\widetilde{\psi}_{[\mu}\gamma_{\nu\rho}\psi_{\sigma]}). \tag{3.6}$$

Despite the GPM spinors employed, other properties such as Fierz rearrangement formulae remain the same as in M11D [6]. For example [6, 10]

$$\frac{1}{8}(\gamma^{\mu\rho\sigma\tau\lambda\omega})_{\alpha(\beta|}(\gamma_{\lambda\omega})_{|\gamma\delta)} - \frac{1}{8}(\gamma_{\lambda\omega})_{\alpha(\beta|}(\gamma^{\mu\rho\sigma\tau\lambda\omega})_{|\gamma\delta)} + \frac{1}{4}(\gamma^{\mu\rho\sigma\tau\lambda})_{\alpha(\beta|}(\gamma_{\lambda})_{|\gamma\delta)} - \frac{1}{4}(\gamma_{\lambda})_{\alpha(\beta|}(\gamma^{\mu\rho\sigma\tau\lambda})_{|\gamma\delta)} + 4(\gamma^{[\mu\rho\sigma|})_{\alpha(\beta|}(\gamma^{|\tau]})_{|\gamma\delta)} \equiv 0. \tag{3.7}$$

The closure of gauge algebra also holds as in the original supergravity in M11D [6], due to the flipping property (3.2), which is formally the same as that in M11D [6].

4. $N = 1$ GPM supergravity in E4D

The next important supergravity theory is that in E4D, because of its direct relationship with supergravity in Minkowskian 4D (M4D)[4] [11]. The theory of $N = 1$ supergravity in E4D can be obtained either by dimensional reduction [12], or by direct construction, as well. Here we adopt the latter method.

Supergravity in so-called E4D has already been known implicitly for some time. For example, in [9] the gamma matrix $\gamma^4 \equiv i\gamma^0$ has been used, with all the γ-matrices treated as purely Hermitian matrices, as if we were in E4D (x^1, x^2, x^3, x^4) with $x^4 \equiv ix^0$. Also, recently a part of the analysis for spinors in E4D has been performed in [7] in terms of GM or GPM spinors. Additionally, it has also been pointed out in [13] that $N = (0, 1/2)$ supergravity is possible in terms of Weyl spinors in E4D, at the expense of Lorentz invariance. In any case, only symplectic spinors have been known in 'real' E4D [3, 4]. Here, we apply the results in [7] to $N = 1$ supergravity in E4D in terms of GPM spinors without symplectic doubling.

We now have $s = 4, t = 0, s - t = 4$ with $\epsilon = -1, \eta = \pm 1$ in table 1. Similarly to E11D, we have $A = I$,[5] and (1.1) is specified as

$$(\gamma_\mu)^\dagger = \gamma_\mu, \qquad A = I, \qquad \widetilde{\psi} \equiv (\psi^\diamond)^T, \qquad \psi^\diamond = B\psi, \tag{4.1a}$$

$$(\gamma_\mu)^* = (\gamma_\mu)^\diamond = \eta B \gamma_\mu B^{-1}, \qquad B = CA^{-1} = C, \qquad (B^T)^{-1} = -C^{-1}, \tag{4.1b}$$

$$(\gamma_\mu)^T = \eta C \gamma_\mu C^{-1}, \qquad C^T = -C. \tag{4.1c}$$

Accordingly, equation (2.1) is more specified as

$$(\widetilde{\psi}\gamma^{[n]}\chi)^\diamond = \eta^n(\widetilde{\psi}\gamma^{[n]}\chi) = \begin{cases} (\widetilde{\chi}\gamma^{[n]}\psi) & (\text{for } \eta = +1), \\ (-1)^n(\widetilde{\chi}\gamma^{[n]}\psi) & (\text{for } \eta = -1), \end{cases} \tag{4.2a}$$

$$(\widetilde{\psi}\gamma^{[n]}\chi) = (-1)^{n(n-1)/2}\eta^n(\widetilde{\chi}\gamma^{[n]}\psi) = \begin{cases} (-1)^{n(n-1)/2}(\widetilde{\chi}\gamma^{[n]}\psi) & (\text{for } \eta = +1), \\ (-1)^{n(n+1)/2}(\widetilde{\chi}\gamma^{[n]}\psi) & (\text{for } \eta = -1). \end{cases} \tag{4.2b}$$

This implies that GPM spinors with $\eta = -1$ in E4D have the same complex-conjugation and flipping properties as the conventional $N = 1$ Majorana spinors in M4D [11]. For example, the closure of gauge algebra with the real translation parameter $\xi^\mu \equiv i(\widetilde{\epsilon}_1\gamma^\mu\epsilon_2) = -i(\widetilde{\epsilon}_2\gamma^\mu\epsilon_1)$ holds only for $\eta = -1$. Therefore, we have in E4D formally the same Lagrangian for GPM $N = 1$ supergravity as in M4D [11].

[4] The 'M4D' here are with the signature $(+, -, -, -)$.

[5] Note that despite the similarity to [9], the latter used $A = \gamma^4 \neq I$. In this sense, the spacetime used in [9] is 'pseudo'-Euclidean 4D (PE4D) different from our presentation here.

Our action $I_{\text{E4D}} \equiv \int d^4x \mathcal{L}_{\text{E4D}}$ is now in terms of GPM gravitino ψ_μ for $\eta = -1$:

$$\mathcal{L}_{\text{E4D}} = -\frac{1}{4} e R(\omega) - \frac{i}{2} e [\widetilde{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu(\omega) \psi_\rho], \tag{4.3}$$

invariant under $N = 1$ GPM supersymmetry

$$\delta_Q e_\mu{}^m = -i(\widetilde{\epsilon}\gamma^m \psi_\mu), \qquad \delta_Q \psi_\mu = +D_\mu(\hat{\omega})\epsilon. \tag{4.4}$$

Since the definitions of $\hat{\omega}_\mu{}^{rs}$, etc are parallel to the previous E11D case, we suppress such redundancies. Also, the couplings of supergravity to other multiplets are parallel to the corresponding M4D cases [11] that we do not write explicitly here.

5. Concluding remarks

In this paper, based on the formulation presented by [7], we have shown that PGM spinors can be used to formulate $N = 1$ supergravity both in E11D and E4D, without conventional $Sp(1)$ symplectic spinors ψ_i. The GM or PGM has the property $(\psi^\diamond)^\diamond = -\psi$ compatible with $B^*B = \epsilon I = -I$ [3, 4] in spacetime dimensions with $2 \leqslant s - t \leqslant 6 \,(\text{mod}\, 8)$.

We next investigated complex conjugations and flipping properties of GM and GPM-spinor bilinears in ${}^\forall d = t + s$ for the case $\epsilon = -1$, as summarized in table 1. These are practically very useful for constructing new supergravity theories in diverse dimensions.

As an example, we gave simple $N = 1$ GPM supergravity in E11D with 128 + 128 physical degrees of freedom, as opposed to the conventional analysis with symplectic doubling [3–5]. Subsequently, we constructed $N = 1$ GPM supergravity in E4D without symplectic doublings, as opposed to the conventional wisdom [3–5]. Even though its Lagrangian and transformation rule are formally the same as $N = 1$ supergravity in M4D [11], we have acquired conceptional progress, because we now have $N = 1$ simple supergravity in E4D in terms of GPM spinors without symplectic doublings [3–5].

We have now three versions for possible spinors in 'E11D':

(i) Those in conventional $N = 1$ supergravity [6, 9] in 'pseudo'-E11D (PE11D) by Wick rotations [14].
(ii) 'Conventional' spinors with symplectic doublings [3, 4, 15, 16]
(iii) GPM spinors [7] in present $N = 1$ supergravity in this paper.

First, the version (i) is the conventional formulation in PE11D [9], obtained as analytic continuation from M11D, with the prescriptions such as $A = \gamma^{11} \neq I$. This version has its advantage that it is related to supergravity in M11D by analytic continuation or Wick rotations [14]. This version (i) is also closely related to quantum field theory of supergravity in M11D [6], because of analytic continuation from M11D. Second, the version (ii) with symplectic doublings [3, 4] is the most 'counter-intuitive', because even the spinorial degrees of freedom double, when comparing with the conventional M11D supergravity. On the other hand, the version (iii) with GPM spinors with the same spinorial degrees of freedom looks more advantageous than (ii), providing an alternative version to the version (i) in PE11D. Finally, the version (iii) plays a crucial role, when investigating recently developed non-anti-commutative supersymmetry or supergravity [1, 13].

Euclidean spacetime for (supersymmetric) field theories is of primary interest in the context of path-integral approach to quantum field theory. Path integrals use the standard complex conjugation on Grassmann algebra. The formulation in this paper is already in Euclidean spacetime. A question arises as to the validity of this Euclidean field theory, when analytically continued to Minkowski spacetime. We believe that this can be done consistently, once the relevant conjugation operator and other relevant operations are defined on the Grassmann algebra.

There are at least three methods of achieving this goal. The first method is to construct the relevant conjugation operator by hand. The second one is to define the conjugation operator with respect to a continuous parameter as a generalized Wick rotation [16]. Choosing the specific values of the parameter will then define the theory either in Euclidean or Minkowskian space. The third method is to define the conjugation operator in terms of an external metric $g_{AB}(\theta)$ that depends on the parameter θ, such that with $\theta = 0$ one gets the Minkowski theory and with $\theta = \pi/2$ one gets the Euclidean theory [17]. In our case, we need to use the above-mentioned method(s) to construct the complex-conjugation operator necessary to go from E11D to M11D and *vice versa*.

Admittedly, we have not performed any of these constructions in this paper, because this formulation had recently been started mainly in the context of non-(anti)commutative coordinates [1, 2]. We hope to address this problem in a future publication.

In any case, it is advantageous to have formulations within E11D directly, where the same degrees of freedom are established as the corresponding M11D, also from the viewpoint of quantum field theory. It is more likely that E11D theory with GPM spinors is closely related to PE11D theory or M11D itself, maintaining the same degrees of freedom.

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[6] Note the change of the author's second name in the published version.